DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

169. Proposed F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

At what rate per cent. must a note be discounted at the end of a quarter of a year in order to produce a discount equivalent to 10% interest for the year?

No solution received.

170. Proposed by J. F. LAWRENCE, A. B., Breckenridge, Mo.

Suppose the market value of 5% bank stock to be 11 1-9% higher than 8% corporation bonds; I realize 8% on my investment, and my income from each is \$180. What did I invest in each?

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

$11\frac{1}{9}\% = \frac{1}{9}$. $\therefore$ The 5% stock costs $\frac{10}{9}$ as much per share as the 8% stock.

$\therefore \frac{\text{Idvestment in 5\% stock}}{\frac{10}{9}} \times \frac{5}{100} = \text{Investment in 8\% stock} \div \frac{8}{100}$.

$\therefore$ Investment in 5% stock : Investment in 8% stock = 16 : 9.

Total investment = \$360 ÷ .08 = \$4500.

Investment in 5% stock = $\frac{16}{25}$ of \$4500 = \$2880.

Investment in 8% stock = $\frac{9}{25}$ of \$4500 = \$1620.

Also solved by *H. B. LEONARD, B. S.*, and *G. W. GREENWOOD, B. A.*

171. Proposed by JOHN S. ROYER, Editor of The School Visitor, Columbus, Ohio.

A drawer made of inch boards is 8 inches wide, 6 inches deep, and slides horizontally. How far must it be drawn out to put into it a book 4 inches thick, 6 inches wide, and 9 inches long?

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and H. B. LEONARD, B. S.

Let a=length of book, b=its thickness, c=depth of drawer, x=distance it must be drawn out. If we draw out the drawer just far enough to put in the book lengthwise we have from the figure properly drawn, $a : x = c : b$ or $9 : x = 5 : 4$. Therefore $x = 7\frac{1}{5}$ inches.

ALGEBRA.

183. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Find the condition that $x : y : z$ may be real, given that $\Sigma ax^2 = \Sigma aax = 0$ and $\Sigma aa^2 = 1$.

Solution by H. B. LEONARD, B. S.

From $ax^2+by^2+cz^2=0=aax+b\beta y+c\gamma z$, $aa^2+\beta^2 b+c\lambda^2=1$, we get

$$x^2(aa^2+c\lambda^2)a+2aab\beta xy+y^2(b\beta^2+c\gamma^2)b=0,$$

$$\frac{x}{y}=-\frac{aab\beta\pm\gamma\sqrt{(-abc)}}{a(1-b\beta^2)}, \qquad \frac{x}{z}=-\frac{aac\lambda\pm\beta\sqrt{(-abc)}}{a(1-c\gamma^2)},$$

$$\frac{y}{z}=-\frac{b\beta c\gamma\pm a\sqrt{(-abc)}}{b(1-aa^2)}.$$

Assuming a, b, c, a, β, γ to be real, then in order that $x:y:z$ may be real, $\sqrt{(-abc)}$ must be real. From $ax^2+by^2+cz^2=0$, it is clear that a, b, c can not all have the same sign and hence we must have one of the quantities a, b, c negative and the other two positive.

184. Proposed by J. A. CALDERHEAD, B.Sc., Professor of Mathematics, Curry University, Pittsburg, Pa.

If m rows, viz., the h_1th, h_2th, ..., h_mth, be transferred so as to become the 1st, 2nd, ..., mth, without altering the relative positions of the remaining rows, and that n columns, viz., the k_1th, k_2th, ..., k_nth, be similarly transformed the determinant thus obtained is the same as the original or differs from it only in sign according as $h_1+h_2+\ldots\ldots+h_m-\frac{1}{2}m(m+1)+k_1+k_2+\ldots\ldots+k_n-\frac{1}{2}n(n+1)$ is odd or even. [*Muir.*]

Solution by G. W. GREENWOOD, B. A. (Oxon), G. B. M. ZERR, A. M., Ph. D., and H. B. LEONARD, B. S.

In transferring the pth row (or column) to the qth row (or column) there are $p-q$ interchanges of adjacent rows (or columns) and therefore $p-q$ changes of sign. Hence, in the given example, there are

$$(h_1-1)+(h_2-2)+\ldots\ldots+(h_m-m)+(k_1-1)+(k_2-2)+\ldots\ldots+(k_n-n),$$
$$i.\ e.,\ h_1+h_2+\ldots\ldots+h_m-\tfrac{1}{2}m(m+1)+k_1+k_2+\ldots\ldots+k_n-\tfrac{1}{2}n(n+1)$$

changes of sign, and the determinant is unaltered in value, or differs only in sign, according as this value is *even* or *odd*; not odd or even as stated.

185. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Without introducing radicals, eliminate x and y from the equations (1) $ax^2+bx+c=0$, (2) $ay^2+by+d=0$, and (3) $ax^2y^2+bxy+e=0$.

I. Solution by H. F. MacNEISH, A. B., Instructor in Mathematics, University High School, Chicago, Ill., G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and J. E. SAUNDERS, Hackney, Ohio.

The eliminant of (1) and (3) is

$$\begin{vmatrix} a, & b, & c, & 0 \\ 0, & a, & b, & c \\ ay^2, & by, & e, & 0 \\ 0, & ay^2, & by, & e \end{vmatrix}=0$$